

Second Semester B.E. Degree Examination, June/July 2013
Engineering Mathematics – II

Time: 3 hrs.

Max. Marks:100

Note: 1. Answer any FIVE full questions, choosing at least two from each part.
2. Answer all objective type questions only in OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1 a. Choose the correct answers for the following: (04 Marks)

i) A differential equation of the first order but of higher degree, solvable for y, has the solution as

- A) $F(x, y, c) = 0$ B) $F(x, c_1, c_2) = 0$
C) $F(x, p, c) = 0$ D) $F_1(x, y, c)F_2(x, y, c) = 0$

ii) If $c^2x^2 + 1 = 2cy$ is the general solution of a differential equation then its singular solution is

- A) $y = x$ B) $y = -x$ C) both (A) and (B) D) none of these

iii) The general solution of the differential equation $p = \log(px - y)$ is

- A) $y = px + e^p$ B) $y = px - e^p$ C) $y = px - e^c$ D) $y = cx - e^c$
A) p B) y C) x

b. Solve $xp^2 + p(3x^2 - 2y^2) - 6xy = 0$. (05 Marks)

c. Solve $y = p \sin p + \cos p$. (05 Marks)

d. Solve $y^2 \log y = xyp + p^2$. (06 Marks)

2 a. Choose the correct answers for the following: (04 Marks)

i) $\frac{1}{f(D)}(e^{3x}x^2) =$

- A) $e^{3x} \frac{1}{f(D-3)}x^2$ B) $e^{3x} \frac{1}{f(D-3)}x^2$ C) $x^2 \frac{1}{f(D-3)}e^{3x}$ D) $x^2 \frac{1}{f(D+3)}e^{3x}$

ii) The roots of auxiliary equation of $(D^4 + 2D^3 - 5D^2 - 6D)y = 0$ are

- A) -1, -1, 2, -3 B) 0, -1, 2, -3 C) 0, 1, -2, 3 D) 0, -1, 2, 3

iii) The particular integral of $(-D + 2)y = 3e^{2x}$ is

- A) $\frac{x^3 e^{2x}}{3}$ B) $\frac{x^3 e^{2x}}{2}$ C) $\frac{x^3 e^{2x}}{2}$ D) $-\frac{x^3 e^{2x}}{6}$

iv) If $\frac{dx}{dt} - 2y = 0$, $\frac{dy}{dt} - 2x = 0$ then y is a function of

- A) e^{2t} and e^{-2t} B) e^{2it} and e^{-2it} C) e^t and e^{-2t}

b. Solve $(D^3 - 6D^2 + 11D - 6)y = 2^x + \cos 2x$. (05 Marks)

c. Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$. (05 Marks)

d. Solve $\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0$, $\frac{dy}{dt} + 5x + 3y = 0$. (06 Marks)

(04 Marks)

D) none of these

C) $\frac{2s}{s^2 - 4}$

D) $\cot^{-1}(s - 1)$

C) $\tan^{-1} s$

D) none of these

C) $f(s) - f(0)$

D) $e^{-2s} \sin 4$

C) $e^{-4s} \sin 2$

(05 Marks)

b. Prove that $L(t^n) = \frac{n!}{s^{n+1}}$, if n is a positive integer.

(05 Marks)

c. Find $L\left(\frac{e^{-t} \sin t}{t}\right)$ and hence find $\int_0^\infty \frac{e^{-t} \sin t}{t} dt$.

(06 Marks)

d. Express: $f(t) = t - 1, 1 < t < 2$
 $= -t - 3, 2 < t < 3$
 $= 0, \text{ otherwise}$
in terms of unit step function and hence find $L(f(t))$.

(06 Marks)

(04 Marks)

8 a. Choose the correct answers for the following:

i) $L^{-1}(s^{-5/2}) =$
A) $\frac{2t^{3/2}}{\sqrt{\pi}}$ B) $\frac{4t^{3/2}}{3\sqrt{\pi}}$ C) $\frac{8t^{3/2}}{15\sqrt{\pi}}$ D) none of these

D) none of these

(06 Marks)

ii) $L^{-1}(\bar{f}(s) \cdot \bar{g}(s)) =$

- A) $f(t) \cdot g(t)$ B) $\int_0^t f(u)g(t-u)du$ C) $\int_0^t f(t-u)g(u)du$ D) either (B) or (C)

iii) $L^{-1}\left(\frac{1}{s^2 + 5}\right) =$

- A) $\frac{1}{5} \sin \sqrt{5}t$ B) $\frac{1}{\sqrt{5}} \sin \sqrt{5}t$ C) $\frac{1}{\sqrt{5}} \sin \sqrt{5}t$ D) $\sin \sqrt{5}t$

iv) $L^{-1}\left(\int_0^s f(s) ds\right) =$

- A) $t f(t)$ B) $\frac{f(t)}{t}$ C) $\frac{f(s)}{s}$ D) none of these

(05 Marks)

b. Find $L^{-1}\left\{\log \frac{s+1}{s-1}\right\}$.

(05 Marks)

c. Find $L^{-1}\left[\frac{1}{4s^2 - 9}\right]$ by using convolution theorem.

d. Solve by using Laplace transformation $y'' + 2y' - y^2 - 2y = 0$ where $y = 1, \frac{dy}{dt} = 2 = \frac{d^2y}{dt^2}$ at $t = 0$. (06 Marks)

3 a. Choose the correct answers for the following:

(04 Marks)

- i) The complementary function of $x^2y'' + 4xy' + 2y = e^x$ is
 - A) $c_1e^{-x} + c_2e^{-2x}$
 - B) $c_1(-x) + c_2(-2x)$
 - C) $c_1e^{-x} + c_2e^{2x}$
 - D) $\frac{5x}{x} + \frac{c_2}{x^2}$
- ii) If $y = u(x) \cdot 1 + v(x) \cdot e^{2x}$ is a particular integral of $y'' + y = \operatorname{cosec} x$ in the method of variation of parameters then $v(x) =$
 - A) e^{2x}
 - B) e^{2x}
 - C) e^{2x}
 - D) $-e^x$
- iii) The roots of the auxiliary equation of the transformed equation of:
 - A) $3, -1$
 - B) $-3, 1$
 - C) $12, -4$
 - D) none of these
- iv) Indicial equation is related to
 - A) singular point
 - B) regular singular point
 - C) ordinary point
 - D) none of these

b. Solve $(D^2 + 1)y = \tan x$ by method of variation of parameters. (05 Marks)

c. Solve $x^2y'' - xy' + 2y = x \sin(\log x)$. (05 Marks)

d. Solve $(1 + x^2)y'' + xy' - y = 0$ in series solution. (06 Marks)

4 a. Choose the correct answers for the following:

(04 Marks)

- i) $z = (x - a)^2 + (y - b)^2$, a and b are arbitrary constants, is a solution of
 - A) $z = 2p^2 + 2q^2$
 - B) $4z = p^2 + q^2$
 - C) $p = 2(x - a)$
 - D) $q = 2(y - b)$
- ii) For $z = (x + a)(x + b)$, $z = 0$ is a
 - A) singular solution
 - B) general solution
 - C) particular solution
 - D) complete solution
- iii) Suitable set of multipliers to solve $(y^2 + z^2)p + xyq = zx$.
 - A) $0, 1, 1$
 - B) $x, -y, -z$
 - C) $1, -\frac{y}{x}, -\frac{z}{x}$
 - D) all of these
- iv) Taking $Z = X(x) \cdot Y(y)$ is a solution of a partial differential equation then this procedure is called
 - A) separation of variables
 - B) Lagrange's method
 - C) separation of variables
 - D) Partial separation of variables

b. Form a partial differential equation by eliminating arbitrary function, from the relation

$$z = f\left(\frac{xy}{z}\right). \quad (05 \text{ Marks})$$

c. Solve $xp - yq = y^2 - x^2$. (05 Marks)

d. Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables. (06 Marks)

5 a. Choose the correct answers for the following:

(04 Marks)

- i) $\int_0^{1-x} \int_0^{1-x-y} (x^2 - y^2) dx dy =$
 - A) 0
 - B) $\frac{1}{12}$
 - C) $\frac{1}{6}$
 - D) none of these

PART - B

5 a. ii) $\int_0^1 \int_0^{2-2x} \int_0^{2-2x-x} dz dy dx =$

- A) 3
- B) 2
- C) 1
- D) none of these

iii) $\int_0^1 \int_0^1 \left[\log\left(\frac{1}{x}\right) \right]^{\frac{1}{2}} dx =$

- A) $\Gamma\left(\frac{1}{2}\right)$
- B) $\Gamma\left(\frac{3}{2}\right)$
- C) $\Gamma\left(\frac{5}{2}\right)$
- D) none of these

iv) $\int_0^{x/2} \int_0^{x/2} \cos^m x \, dx =$

- A) $\frac{1}{2} \beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$
- B) $\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$
- C) $\frac{1}{2} \beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$
- D) $2\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$

b. Change into polar coordinates and evaluate $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{\cos \theta} e^{-x^2-y^2} dy dx$ (05 Marks)

c. Evaluate $\int_{-b}^b \int_{-a}^a \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$. (05 Marks)

d. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (06 Marks)

6 a. Choose the correct answers for the following:

(04 Marks)

- i) Which theorem gives a relation between surface integral and volume integral?
 - A) Green's
 - B) Stoke's
 - C) Divergence
 - D) None of these
- ii) If S is any closed surface enclosing the volume, V then by Divergence theorem, the value of $\int_S \sqrt{R} \cdot dS$ is
 - A) 0
 - B) 1
 - C) 2
 - D) 3
- iii) The work done by the force $\vec{F} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ moves a particle from $(0, 0, 0)$ to $(2, 1, 1)$ along the curve $x = t^2, y = t, z = 0$ is
 - A) $3t^2$
 - B) 0
 - C) 1
 - D) none of these
- iv) If S is any closed surface enclosing the volume, V then by Divergence theorem, the value of $\int_S \sqrt{R} \cdot dS$ is
 - A) V
 - B) $2V$
 - C) $3V$
 - D) none of these

b. Use Green's theorem to evaluate $\iint_C [(y - \sin x) dx + \cos x dy]$ where C is enclosed by $y = 0, x = \frac{\pi}{2}, y = \frac{2}{\pi} x$. (05 Marks)

c. Use Stoke's theorem to evaluate $\int_S \operatorname{curl} \vec{F} \cdot d\vec{S}$ where $\vec{F} = y\mathbf{i} + (x - 2xz)\mathbf{j} - xy\mathbf{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy -plane. (05 Marks)

d. By transforming to a triple integral, evaluate $\int_S x^2 dy dz + x^2 y dz dx + x^2 z dx dy$ where S is the closed surface bounded by the planes $z = 0, z = b$ and the cylinder $x^2 + y^2 = a^2$. (06 Marks)